

Convergence of Mimetic Finite Difference Method for Diffusion Problems on Polyhedral Meshes

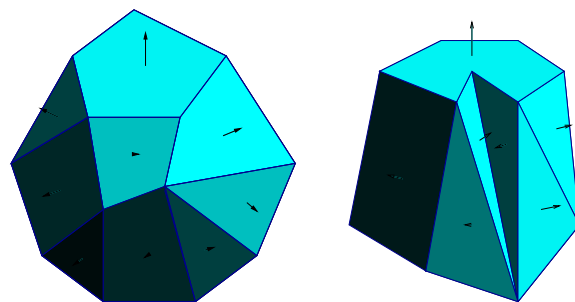
Franco Brezzi, brezzi@imati.cnr.it
Konstantin Lipnikov, lipnikov@lanl.gov
Mikhail Shashkov, shashkov@lanl.gov

In many applications meshes with general type elements are more preferable than standard tetrahedral or hexahedral meshes. Allowing arbitrary shape for a mesh element provides greater flexibility in the mesh generation process, especially in the regions where the geometry is extremely complex. For example, in a reservoir simulation, the thinning or tapering out ("pinching out") of geological layers can be easily modeled with pentahedrons and prisms.

The predictions and the insights gained from numerical simulations on polyhedral meshes are trustworthy if only if the reliable and accurate discretization methods have been used. An example of such a method is the mimetic finite difference (MFD) method [1]. For the linear diffusion problem with a sufficiently smooth solution the MFD method exhibits the second-order convergence rate for the fluid pressure and the first-order convergence rate for the fluid velocity.

Similar convergence rates are observed in other lower order discretization methods under different assumptions on the computational mesh (e.g., finite element methods on simplicial meshes). The main advantage of the MFD method is its flexibility. We have proved in [2] that the method converges on meshes with degenerate and non-convex elements with flat faces (see the figure below). The irregular elements appear in mesh refinement methods (with hanging nodes), in moving mesh methods and in non-matching mesh methods.

For the linear diffusion problem, the MFD method mimics the Gauss divergence theorem, the symmetry between the gradient and diver-



Two possible elements and the normal to their faces.

gence operators, and the null spaces of the involved operators. Therefore, it produces the discretization scheme which is locally conservative, exact for uniform flow, and results in symmetric positive definite coefficient matrix. The above properties are hold for the full diffusion tensor.

The original proof of the convergence of the MFD method on quadrilateral and triangular meshes was based on establishing a relationship with a mixed finite element method. In [2] we developed a new methodology which has both theoretical and practical value. It provides a constructive way for extending the MFD method to polyhedral meshes with curved faces. At the moment, a curved face may be approximated by flat triangles which still gives a discrete problem with smaller number of unknowns relative to a tetrahedral partition.

Acknowledgements

Los Alamos Report LA-UR-04-5756. Funded by the Department of Energy under contract W-7405-ENG-36. The authors acknowledge the partial support from the DOE Office of Science's Advanced Scientific Computing Research (ASCR) program in Applied Mathematical Sciences and from the Advanced Simulation and Computing (ASC) Program.

References

- [1] Y. Kuznetsov, K. Lipnikov, M. Shashkov. The mimetic finite difference method on polygonal meshes for diffusion-type problems. To appear in *Comp. Geosciences*, 2004.
- [2] F. Brezzi, K. Lipnikov, M. Shashkov. Convergence of mimetic finite difference method for diffusion problems on polyhedral meshes. Los Alamos National Laboratory, Report LA-UR-04-5756, submitted to *SIAM J. Numer. Anal.*, 2004.